Turning Points

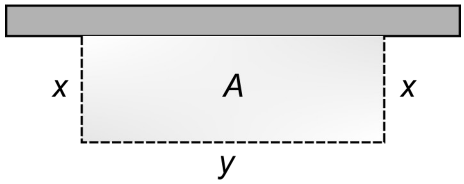
1. Calculate the coordinates (x, y) of the turning points on each of the following functions.
Find whether each is a maximum, minimum, or point of inflection.

   (a) $y = 5x^2 - 10x + 7$

   (b) $y = x^3 - 6x^2 + 9x$

   (c) $y = \frac{x^3}{3} + \frac{3x^2}{2}$

2. A 100 m length of fencing is used on three sides to enclose a rectangular plot of land. The fourth side is formed by an existing wall.
This question is about finding the maximum area that can be enclosed by the fence.



   (a) Let the length parallel to the wall be *y*, and the length of the other two sides of the rectangle be *x*. Write an expression connecting *x* and *y* and 100.

   (b) Write an expression for the area enclosed, *A*, in terms of *x* and *y*.

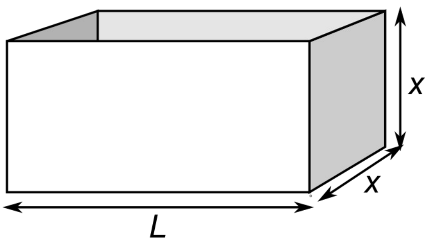
   (c) Rearrange the expression from part (a) to make *y* the subject.

   (d) Substitute this value of y into the expression from part (b), to get *A* in terms of *x*.

   (e) By differentiation, find the value of *x* when *A* is a maximum.

   (f) Calculate the largest area that can be enclosed using this 100 m of fencing, by using this value of *x*.

3. A large rectangular tank, without a lid, is made from 54 m^2 of sheet metal.
The tank has a rectangular base, length *L*
The end faces are squares, of side *x*.
This question is about finding the maximum volume possible for the tank.



   (a) Write an expression for volume, *V*, of the tank, in terms of *L* and *x*.

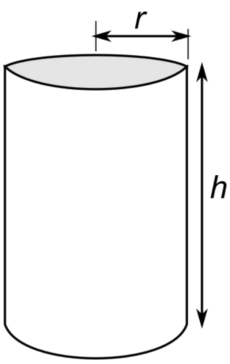
   (b) Write an expression for the area, *A*, of metal used, in terms of *L* and *x.* *(there are 5 sides)*

   (c) Rearrange the expression from part (b) to get *L*, and substitute into the volume expression from part (a), to get an equation for V, in terms of *x*. *(Remember that A = 54)*

   (d) Calculate the value of *x* for which *dV/dx* is zero and *V* is a maximum.

   (e) Calculate the maximum volume of the tank using 54 m^2 of metal, using the equation in part (c).

4. This question is about finding the smallest area of sheet aluminium need to make a drinks can of volume 375 ml.
The can is a cylinder, capped at both ends, radius r, height h.



(a) Write an expression for the surface area, A, of the can, in terms of r and h.

(b) Write an expression for the volume, V, of the can, in terms of r and h.

(c) Rearrange the equation from part (b), to get h, then substitute for h in the equation from part (a), to get A in terms of r.
(Remember $V = 375$)

(d) Calculate the value of r for which dA/dr is zero and A is a minimum.

(e) Find the smallest area of aluminium needed, using the equation from part (c).

Answers:

1. (1, 2), minimum;
   (2, 8), inflection;
   (0, 0), minimum & (–3, 4.5), maximum.
2. $x = 25$ m, $A = 1250$ m^2
3. $x = 3$, $V = 36$ m^3
4. $r = 3.91$ cm, $A = 288$ cm^2