Differentiation – chain rule

The chain rule is also called ‘function of a function’.

e.g. $(\sin x)^3$ is … the cube of … the sine of … *x*

$e^{(x^2)}$ is … *e* to the power of … the square of … *x*

Example 1

$y = (\sin x)^3$ … ‘define’ a new variable $z = (\sin x)$

We now have: $z = (\sin x)$, $y = z^3$

and: $\frac{dz}{dx} = \cos x$, $\frac{dy}{dz} = 3z^2$

Apply chain rule: $\boxed{\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}}$

$\frac{dy}{dx} = 3z^2 \times \cos x$

but z is not part of the original expression, so we must substitute …

$\frac{dy}{dx} = 3(\sin x)^2 \times \cos x = 3\sin^2 x \cos x$

Example 2

Calculate $\frac{dy}{dx}$, when $y = e^{(x^2)}$

Define a new variable, $z = x^2$

We have:	$z = x^2$	$y =$
	$\frac{dz}{dx} =$	$\frac{dy}{dz} =$
Apply chain rule: $\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx} =$ now substitute for *z*:		

Practice

Find $\frac{dy}{dt}$ when

(a) $y = \sin(t^2)$ (b) $y = \sqrt{(t+1)}$ (c) $y = e^{2t^2}$

(d) $y = \cos(3t^2)$ (e) $y = \sin(2t+1)$ (f) $y = e^{4t-3}$

Answers:

(a) $2t\cos t^2$, (b) $\frac{1}{2\sqrt{(t+1)}}$, (c) $4te^{2t^2}$,

(d) $-6t\sin(3t^2)$, (e) $2\cos(2t+1)$, (f) $4e^{4t-3}$.

Differentiation Revision Questions

Find

(a) $\frac{d}{d\theta}\cos(4\theta)$ (b) $\frac{d}{dt}e^{-7t}$ (c) $\frac{d}{dz}\ln(5z)$ (d) $\frac{d}{dx}(2x-x^2)$

(hint: (e) and (f) do not need product/quotient/chain rules)

(e) $\frac{d}{dy}\sqrt{2y}$ (f) $\frac{d}{dx}\frac{(x-x^2)}{x^3}$ (g) $\frac{d}{dx}x\sin x$ (h) $\frac{d}{d\theta}\sin\theta.\cos\theta$

(i) $\frac{d}{dx}\frac{x}{(1-x^2)}$ (j) $\frac{d}{dx}\frac{\sin x}{x}$

Answers:

(a) $-4\sin(4\theta)$, (b) $-7e^{-7t}$, (c) $\frac{1}{z}$, (d) $2-2x$,

(e) $\frac{1}{\sqrt{2y}}$, (f) $-\frac{2}{x^3}+\frac{1}{x^2}$, (g) $\sin x+x\cos x$, (h) $\cos^2\theta-\sin^2\theta$,

(i) $\frac{1+x^2}{(1-x^2)^2}$, (j) $\frac{x\cos-\sin x}{x^2}$