Differentiation – products and quotients

$\sqrt{x}(\cos x)$ is a product of two functions of x.

$\dfrac{x^2}{\sin x}$ is a quotient of two functions of x.

If you want to differentiate a product or a quotient, you need the product rule or quotient rule.

Product

Note: Although $x^2(1 + x)$ is a product, you do not need a special rule.
You just multiply to get a polynomial.

Example: To find $\dfrac{d}{dx}\left(\sqrt{x}(\cos x)\right)$

write the two functions as, e.g., u and v: $\begin{aligned} u &= \sqrt{x} \\ v &= \cos x \end{aligned}$

Apply the product rule: $\boxed{\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}}$

e.g. Find $\dfrac{d}{dx}\left(\sqrt{x}(\cos x)\right)$

$u =$ $\qquad\qquad$ $\dfrac{du}{dx} =$

$v =$ $\qquad\qquad$ $\dfrac{dv}{dx} =$

$u\dfrac{dv}{dx} + v\dfrac{du}{dx} =$

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Quotient

Note: Although $\dfrac{(6x^2+1)}{2x}$ is a quotient, you do not need a special rule.
You just divide to get a polynomial.

Example: To find $\dfrac{d}{dx}\left(\dfrac{x^2}{\sin x}\right)$

write the two functions as, e.g., u and v: $u = x^2$, $v = \sin x$

Apply the quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v\dfrac{du}{dx} - u\dfrac{dv}{dx}\right)}{v^2}$$

e.g. Find $\dfrac{d}{dx}\left(\dfrac{x^2}{\sin x}\right)$

$u =$ $\qquad$ $\dfrac{du}{dx} =$

$v =$ $\qquad$ $\dfrac{dv}{dx} =$

$$\frac{\left(v\dfrac{du}{dx} - u\dfrac{dv}{dx}\right)}{v^2} =$$

Practice:

1. $y = x^2 \sin x$
2. $y = e^x \cos x$
3. $y = \dfrac{(x+3)}{(x-2)}$
4. $y = 7x\ln(x)$
5. $y = 3x^2 \sin 2x$
6. $y = \dfrac{\sin x}{3x^2}$

Answers:

1. $x^2\cos x + 2x\sin x$
2. $e^x(\cos x - \sin x)$
3. $-\dfrac{5}{(x-2)^2}$
4. $7(1+\ln x)$
5. $6x(\sin 2x + x\cos 2x)$
6. $\dfrac{x\cos x - 2\sin x}{3x^3}$