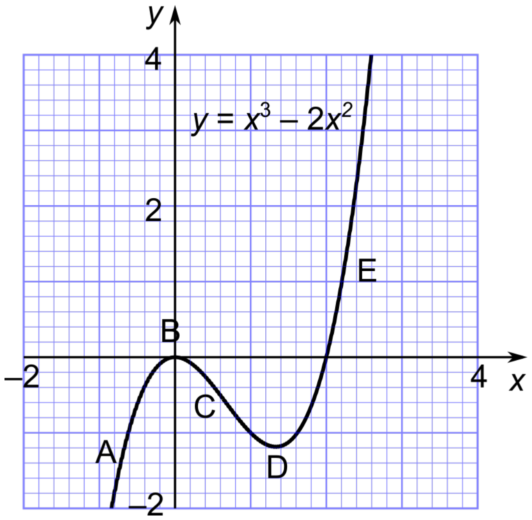
Turning Points

Sometimes you want to find the maximum or minimum value of a function, for example to:

maximise the volume of a container using a given area of sheet material,
minimise the cost of a product which depends on a number of variables.

Differentiation can find maxima and minima.

Example 1



point	gradient (–ve / 0 / +ve)
A	
B	
C	
D	
E	

Points where the function is maximum or minimum are called **turning points** (or stationary points).

$\frac{dy}{dx} = 0$ at a maximum or minimum.

$y = x^3 - 2x^2 \ldots \frac{dy}{dx} =$

$\frac{dy}{dx} = 0$ ... so ...

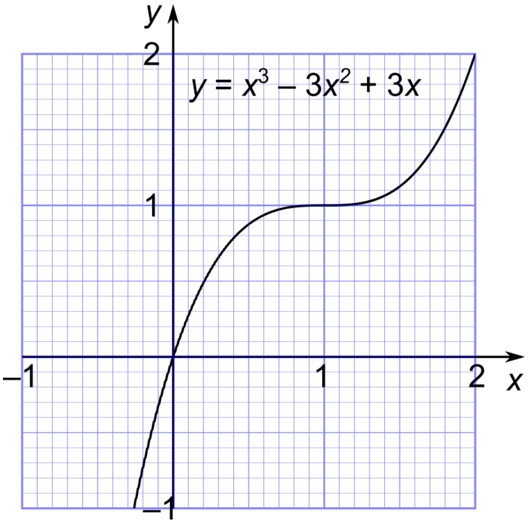
$x =$ .... or $x =$ ....

substitute the values of *x* into the original $y = x^3 - 2x^2$ to find the two values of *y*:

$y =$ .... $y =$ ....

Another possibility where $\frac{dy}{dx} = 0$:

Example 2



$$y = x^3 - 3x^2 + 3x$$

$$\frac{dy}{dx} =$$

when $\frac{dy}{dx} = 0$

$$\text{........}\, x^2 - \text{........}\, x + \text{........} = 0$$

$$(\quad - \quad)(\quad - \quad) = 0$$

$x = \text{........}$ $\quad y = x^3 - 3x^2 + 3x =$

This turning point is called a **point of inflection**.

Using the second derivative

If you differentiate $\frac{dy}{dx}$ a second time, you get $\frac{d^2y}{dx^2}$

('dee-two-y by dee-x-squared')
This is the second derivative of y, which can be written y".

Example 1	$y = x^3 - 2x^2$ $\frac{dy}{dx} = 3x^2 - 4x$ $\frac{d^2y}{dx^2} =$	when $x=0$, $\frac{d^2y}{dx^2} =$ when $x=\frac{4}{3}$, $\frac{d^2y}{dx^2} =$
Example 2	$y = x^3 - 3x^2 + 3x$ $\frac{dy}{dx} = 3x^2 - 6x + 3$ $\frac{d^2y}{dx^2} =$	when $x=1$, $\frac{d^2y}{dx^2} =$

if $\frac{d^2y}{dx^2} > 0$	if $\frac{d^2y}{dx^2} = 0$	if $\frac{d^2y}{dx^2} < 0$