Exponential Growth and Decay

Exponential growth/decay is where a quantity changes by the same ***fraction*** for every time period.
e.g.

number of cases in a pandemic doubles every week,
money invested increases by a factor of 1.02 every year (2% interest),
activity of a sample of U-238 halves every 4,500,000,000 years,
charge on a capacitor in a circuit falls to 0.95 (5% decrease) every second.

General equations

growth: $N = N_0 e^{kt}$

decay: $N = N_0 e^{-kt}$

where:
N = quantity (pandemic cases / money / activity / charge)
N_0 = original quantity
k = constant showing speed of growth (or decay, if –ve)
t = time

If we take logs (to base e) of e.g. growth equation:

$$N = N_0 \times e^{kt}$$

multiplying numbers means adding logs:

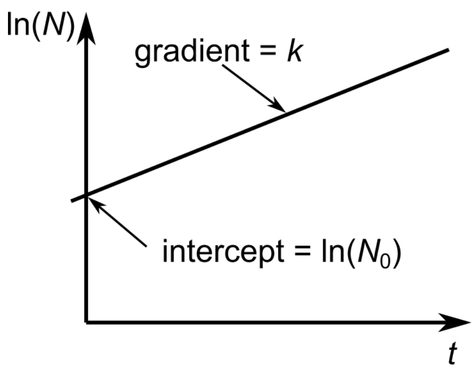
$$\ln(N) = \ln(N_0) + \ln(e^{kt})$$

ln and e^x are inverse functions, so: $\ln(e^x) = x$

$$\ln(N) = \ln(N_0) + kt$$

this is a straight line: $y \quad = \quad C \; + \; m\,x$

where: $\ln(N_0) \; = \;$ intercept on ln axis
$k \; = \;$ gradient



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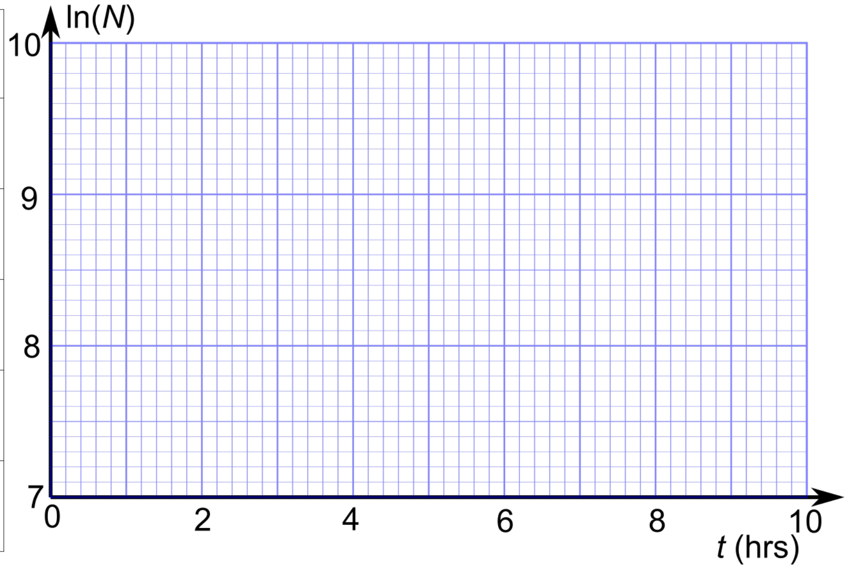
Example

N is number of bacteria counted in an experiment at time *t*.

t (hrs)	*N*	ln(*N*)
2	2090	
4	3640	
6	6330	
8	11030	
10	19200	



To find the initial number, when the experiment started, N_0:

Intercept on ln(*N*)-axis = ...

This is ln(N_0), so $N_0 = e^{\text{intecept}}$ = ...

Gradient = $k = \frac{\Delta y}{\Delta x} = \frac{\qquad}{\qquad}$ = ...

(7.1, 1200, 0.28)